

Absence of Unruh effect in polymer quantization

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(Dated: November 10, 2014)

Unruh effect is a landmark prediction of standard quantum field theory in which Fock vacuum state appears as a thermal state with respect to an uniformly accelerating observer. Given its dependence on trans-Planckian modes, Unruh effect is often considered as an arena for exploring a candidate theory of quantum gravity. Here we show that Unruh effect disappears if, instead of using Fock quantization, one uses polymer quantization or loop quantization, the quantization method used in loop quantum gravity. Secondly, the polymer vacuum state remains a vacuum state even for the accelerating observer in the sense that expectation value of number density operator in it remains zero. Finally, if experimental measurement of Unruh effect is ever possible then it may be used either to verify or rule out a theory of quantum gravity.

PACS numbers: 04.62.+v, 04.60.Pp

Introduction. – A challenging problem for any theory of quantum gravity is to make physical predictions which are within the reach of current experiments either directly or indirectly. This problem originates from the fact that the energy scale accessible even by modern experiments are too small compared to the scale of quantum gravity namely the *Planck scale*. On the other hand, for trans-Planckian modes, such a theory often makes significantly different physical predictions, as compared to those from standard *quantum field theory* and *general relativity*. For low-energy modes, such Planck-scale corrections are usually very small. Therefore, in order to confront a theory that modifies mainly Planck scale physics, one is often forced to look for physical phenomena which are dependent on the trans-Planckian modes yet may have physical implications in a relatively low energy regime.

Unruh effect [1–3] is considered to be an intriguing consequence of applying standard quantum field theory in a curved background [4]. In particular, an *uniformly accelerating* observer finds Fock vacuum state to be a *thermal state* instead of being a zero particle state. In order to obtain this thermal spectrum, as seen by the accelerating observer, one needs to include contributions even from the trans-Planckian modes, as seen by an *inertial observer*. This particular feature makes Unruh effect to be a potentially important arena for understanding and exploring the implications of Planck-scale physics [5].

Polymer quantization [6, 7], the quantization method used in loop quantum gravity [8–10], differs from Schrodinger quantization in several important ways when applied to a mechanical system. Firstly, apart from *Planck constant* \hbar , it contains a new dimension-full parameter. In the context of quantum gravity, this new scale would correspond to *Planck length* $L_p = \sqrt{\hbar G/c^3}$, where G is Newton’s constant of gravitation and c is the

speed of light. Secondly, instead of both, only one of the position and momentum is represented directly as an *elementary operator* in the kinematical Hilbert space. The second operator is represented as exponential of its classical counterpart. These features together with a *distinct* kinematical inner product make polymer quantization unitarily *inequivalent* to Schrodinger quantization [6]. Therefore, in principle, polymer quantization can lead to a different set of results compared to those from Schrodinger quantization.

In the standard derivation of Unruh effect *i.e.* using Fock quantization, the field operator is expressed in terms of *creation* and *annihilation* operators of different Fourier modes. Each of these modes behaves as a *mechanical* system corresponding to a decoupled harmonic oscillator and is quantized using Schrodinger method. In polymer quantization of these modes, the notions of creation and annihilation operators are *not* available. Therefore, we employ here a new method in which Unruh effect is derived using *energy spectrum* of these modes for both Fock and polymer quantizations. Further, we show that Unruh effect is *absent* in the case of polymer quantization.

Rindler spacetime. – The spacetime as seen by an *uniformly accelerating* observer using *conformal* Rindler coordinates $\bar{x}^\alpha = (\tau, \xi, y, z)$ in *natural* units ($c = \hbar = 1$), is described by the metric [11]

$$ds^2 = e^{2a\xi} (-d\tau^2 + d\xi^2) + dy^2 + dz^2 \equiv g_{\alpha\beta} d\bar{x}^\alpha d\bar{x}^\beta, \quad (1)$$

where parameter a denotes the magnitude of *acceleration* 4-vector. For comparison, we consider an *inertial* observer who uses Minkowski coordinate system $x^\mu = (t, x, y, z)$ together with the metric $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$. If the accelerating observer *i.e.* Rindler observer, moves along *+ve* x-axis with respect to the inertial observer then their respective coordinates are related to each other as

$$t = \frac{1}{a} e^{a\xi} \sinh a\tau, \quad x = \frac{1}{a} e^{a\xi} \cosh a\tau. \quad (2)$$

One may note that Rindler spacetime covers only a wedge-shaped region of Minkowski spacetime. This re-

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gion is known as *Rindler wedge*. The y and z coordinates between these two observers are trivially related. So for simplicity we restrict ourselves to an $(1+1)$ dimensional spacetime and denote their respective coordinates as (τ, ξ) and (t, x) for further study.

Massless scalar field. – The dynamics of a massless scalar field in $(1+1)$ dimensional Minkowski spacetime is described by the action

$$S_\Phi = \int dt dx \left[-\frac{1}{2} \sqrt{-\eta} \eta^{\mu\nu} \partial_\mu \Phi(t, x) \partial_\nu \Phi(t, x) \right]. \quad (3)$$

For Rindler observer the scalar field action is $S_\Phi = \int d\tau d\xi \left[-\frac{1}{2} \sqrt{-g} g^{\alpha\beta} \partial_\alpha \Phi(\tau, \xi) \partial_\beta \Phi(\tau, \xi) \right]$. Given the *conformal* structure of Rindler metric $g_{\alpha\beta}$, the action can also be expressed as $S_\Phi = \int d\tau d\xi \left[-\frac{1}{2} \sqrt{-g^0} g^{0\alpha\beta} \partial_\alpha \Phi(\tau, \xi) \partial_\beta \Phi(\tau, \xi) \right]$ where $g_{\alpha\beta}(\tau, \xi) = e^{2a\xi} g_{\alpha\beta}^0(\tau, \xi)$. In other words, for Rindler observer with the coordinates (τ, ξ) , a massless scalar field dynamics can be *equivalently* described using a *flat* metric $g_{\alpha\beta}^0 = \text{diag}(-1, 1)$. Use of this flat metric allows one to perform computations similar to that of an inertial observer. The Hamiltonian corresponding to the action (3) is given by

$$H_\Phi = \int dx \left[\frac{\Pi^2}{2\sqrt{q}} + \frac{\sqrt{q}}{2} q^{ab} \partial_a \Phi \partial_b \Phi \right], \quad (4)$$

where q_{ab} is metric on the *spatial* hyper-surfaces. The Poisson bracket between the field Φ and the conjugate momentum Π is

$$\{\Phi(t, x), \Pi(t, y)\} = \delta(x - y). \quad (5)$$

For Rindler observer, one can write down a similar scalar field Hamiltonian.

Fourier modes. – We define Fourier modes for the scalar field and its momentum with respect to the inertial observer as

$$\Phi = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \tilde{\phi}_{\mathbf{k}}(t) e^{i\mathbf{k}x}, \quad \Pi = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \sqrt{q} \tilde{\pi}_{\mathbf{k}}(t) e^{i\mathbf{k}x}, \quad (6)$$

where $V = \int dx \sqrt{q}$ is the spatial volume. For Minkowski spacetime, the spatial volume would diverge as the space is non-compact. In order to avoid dealing with divergent quantity, we consider a fiducial box of finite volume. In particular, the volume of the box is explicitly chosen as

$$V = \int_{L_{min}}^{L_{max}} dx \sqrt{q} = L_{max} - L_{min} \equiv L. \quad (7)$$

For brevity of notation, we shall skip explicitly writing the limits of the integration. In this context, Kronecker delta and Dirac delta can be expressed as $\int dx \sqrt{q} e^{i(\mathbf{k}-\mathbf{k}')x} = V \delta_{\mathbf{k}, \mathbf{k}'}$ and $\sum_{\mathbf{k}} e^{i\mathbf{k}(x-y)} = V \delta(x-y)/\sqrt{q}$. These expressions together imply that $\mathbf{k} \in \{\mathbf{k}_r\}$ where $\mathbf{k}_r = (2\pi r/L)$ with r being any *non-zero integer*.

In terms of the Fourier modes (6), the Hamiltonian (4) can be reduced to $H_\Phi = \sum_{\mathbf{k}} \mathcal{H}_{\mathbf{k}}$ where Hamiltonian density is

$$\mathcal{H}_{\mathbf{k}} = \frac{1}{2} \tilde{\pi}_{-\mathbf{k}} \tilde{\pi}_{\mathbf{k}} + \frac{1}{2} |\mathbf{k}|^2 \tilde{\phi}_{-\mathbf{k}} \tilde{\phi}_{\mathbf{k}}, \quad (8)$$

and momentum-space Poisson bracket is

$$\{\tilde{\phi}_{\mathbf{k}}, \tilde{\pi}_{-\mathbf{k}'}\} = \delta_{\mathbf{k}, \mathbf{k}'} . \quad (9)$$

Being a *complex-valued* function, each $\tilde{\phi}_{\mathbf{k}}$ has two independent modes. However, the scalar field Φ being a *real-valued* function not all $\tilde{\phi}_{\mathbf{k}}$'s are independent. In particular, reality of the scalar field requires $\tilde{\phi}_{\mathbf{k}}^* = \tilde{\phi}_{-\mathbf{k}}$. We shall impose this reality condition before quantizing these modes.

We define Fourier modes for Rindler observer in a similar manner as $\Phi(\tau, \xi) = (1/\sqrt{\bar{V}}) \sum_{\kappa} \tilde{\phi}_{\kappa}(\tau) e^{i\kappa\xi}$ and $\bar{\Pi}(\tau, \xi) = (1/\sqrt{\bar{V}}) \sum_{\kappa} \sqrt{\bar{q}} \tilde{\pi}_{\kappa}(\tau) e^{i\kappa\xi}$ where the spatial volume $\bar{V} = \int d\xi \sqrt{\bar{q}}$. Here \bar{q} is the determinant of the spatial metric corresponding to the flat spacetime metric $g_{\alpha\beta}^0$. As earlier, the scalar field Hamiltonian for Rindler observer can be reduced to $\bar{H}_\Phi = \sum_{\kappa} \bar{\mathcal{H}}_{\kappa}$ where

$$\bar{\mathcal{H}}_{\kappa} = \frac{1}{2} \tilde{\pi}_{-\kappa} \tilde{\pi}_{\kappa} + \frac{1}{2} |\kappa|^2 \tilde{\phi}_{-\kappa} \tilde{\phi}_{\kappa}, \quad \{\tilde{\phi}_{\kappa}, \tilde{\pi}_{-\kappa'}\} = \delta_{\kappa, \kappa'}. \quad (10)$$

Relation between Fourier modes of two observers. – The scalar field is invariant under coordinate transformation *i.e.* $\Phi(\tau, \xi) = \Phi(t(\tau, \xi), x(\tau, \xi))$. On the other hand, respective canonical momenta $\Pi(t, x) = \partial\Phi(t, x)/\partial t$ and $\bar{\Pi}(\tau, \xi) = \partial\Phi(\tau, \xi)/\partial\tau$ can be related to each other using $\partial\Phi(\tau, \xi)/\partial\tau = (\partial t/\partial\tau)(\partial\Phi(t, x)/\partial t) + (\partial x/\partial\tau)(\partial\Phi(t, x)/\partial x)$. We may recall that in canonical formulation, the spacetime is broken into spatial hyper-surfaces and these hyper-surfaces are labeled by different instances of time. Given an initial field and momentum configuration on a particular hyper-surface, it is possible to dynamically evolve to any other hyper-surface uniquely. Therefore, for simplicity but without loss of generality, we choose the spatial hyper-surface labeled by $t = \tau = 0$ for making comparison between these two observers. At $\tau = 0$, $\partial x/\partial\tau = 0$ and the spatial coordinates x and ξ are related to each other as $a\xi = e^{ax}$. The spatial volume \bar{V} can be expressed as $a\bar{V} = \ln(L_{max}/L_{min})$. The Fourier modes for Rindler observer can be expressed in terms of the modes of the inertial observer as

$$\begin{aligned} \tilde{\phi}_{\kappa} &= \sum_{\mathbf{k}>0} \tilde{\phi}_{\mathbf{k}} F_0(\mathbf{k}, -\kappa) + \sum_{\mathbf{k}>0} \tilde{\phi}_{-\mathbf{k}} F_0(-\mathbf{k}, -\kappa), \\ \tilde{\pi}_{\kappa} &= \sum_{\mathbf{k}>0} \tilde{\pi}_{\mathbf{k}} F_1(\mathbf{k}, -\kappa) + \sum_{\mathbf{k}>0} \tilde{\pi}_{-\mathbf{k}} F_1(-\mathbf{k}, -\kappa), \end{aligned} \quad (11)$$

where $\tilde{\phi}_{\kappa} = \tilde{\phi}_{\kappa}(0)$, $\tilde{\phi}_{\mathbf{k}} = \tilde{\phi}_{\mathbf{k}}(0)$, $\tilde{\pi}_{\kappa} = \tilde{\pi}_{\kappa}(0)$, $\tilde{\pi}_{\mathbf{k}} = \tilde{\pi}_{\mathbf{k}}(0)$. Given $\sqrt{\bar{q}} = 1$, the coefficients F_0, F_1 can be written as

$$F_m(\mathbf{k}, \kappa) = \frac{1}{\sqrt{V\bar{V}}} \int d\xi e^{ma\xi} e^{i\mathbf{k}x + i\kappa\xi}, \quad (12)$$

for $m = 0, 1$. These coefficient functions are analogous to standard *Bogoliubov coefficients*.

Regulation of the coefficient functions. – The definition of $F_m(k, \kappa)$ implies that $F_1(k, \kappa) = (-ia)\partial F_0(k, \kappa)/\partial k$. Clearly, knowing the expression of $F_0(k, \kappa)$ is sufficient. However, the integrand being a pure phase, $F_0(k, \kappa)$ does not *converge* when volume regulators are removed. In order to avoid dealing with formally divergent terms, we introduce a non-oscillatory *regulator* term in the expression of $F_m(k, \kappa)$ as follows

$$F_m^\delta(k, \kappa) = \frac{1}{\sqrt{VV}} \int d\xi e^{ma\xi} e^{ikx + i\kappa\xi} \left[\frac{e^{\delta a\xi}}{d_m} \right], \quad (13)$$

where $d_m = (1 - i\delta ma/\kappa)$. In the limit $\delta \rightarrow 0$, $F_m^\delta(k, \kappa)$ reduces to $F_m(k, \kappa)$. The change of variable $u \equiv |k|x$ in the integration (13) would lead to

$$F_m^\delta(\pm|k|, \kappa) = \frac{a^\beta |k|^{-\beta-1}}{\sqrt{VV} d_m} I_\pm(\beta), \quad (14)$$

where $\beta = (i\kappa/a + \delta + m - 1)$ and $I_\pm(\beta) = \int du e^{\pm iu} u^\beta$. Based on the *sign* of k , the integral can be evaluated by analytic continuation in *upper* or *lower* half of the complex plane respectively as

$$I_\pm(\beta) = e^{\pm i\pi(\beta+1)/2} \Gamma(\beta + 1), \quad (15)$$

where $\Gamma(\beta + 1)$ denotes *Gamma function*. To obtain a *closed-form* expression (15), we have added two extra terms to it, namely $I_{min} \equiv \int_0^{|k|L_{min}} du e^{\pm iu} u^\beta$ and $I_{max} \equiv \int_{|k|L_{max}}^\infty du e^{\pm iu} u^\beta$. Each of these extra terms *identically goes to zero* when volume regulators are removed *i.e.* when the limits $L_{min} \rightarrow 0$ and $L_{max} \rightarrow \infty$ are taken. We note following two useful relations for different arguments of $F_m^\delta(k, \kappa)$ as

$$F_0^\delta(-|k|, \kappa) = e^{\pi\kappa/a - i\delta\pi} F_0^\delta(|k|, \kappa), \quad (16)$$

$$F_1^\delta(\pm|k|, \kappa) = \mp \frac{\kappa}{|k|} F_0^\delta(\pm|k|, \kappa). \quad (17)$$

The requirement that both Poisson brackets $\{\dot{\phi}_\kappa, \tilde{\pi}_{-\kappa}\} = 1$ and $\{\dot{\phi}_k, \tilde{\pi}_{-k}\} = 1$ are simultaneously satisfied, demands $\sum_{k>0} [F_0(k, -\kappa)F_1(-k, \kappa) + F_0(-k, -\kappa)F_1(k, \kappa)] = 1$. Regulated expression of this condition which remains *real-valued* by the choice of d_m , requires

$$\frac{(\kappa/a) |\Gamma(i\kappa/a + \delta)|^2}{2\pi (e^{\pi\kappa/a} - e^{-\pi\kappa/a})^{-1}} = \frac{(a\bar{V})(2\pi/aV)^{2\delta}}{\zeta(1 + 2\delta)}, \quad (18)$$

where $\zeta(1 + 2\delta) \equiv \sum_{r=1}^\infty r^{-(1+2\delta)}$ is *Riemann zeta function*. In the limit $\delta \rightarrow 0$, *lhs* of the equation (18) becomes 1 as $\Gamma(z)\Gamma(1-z) = \pi/\sin \pi z$. Together with *zeta function identity* $\lim_{s \rightarrow 0} [s \zeta(1+s)] = 1$, the equation (18) requires $aL_{min} \simeq 2\pi e^{-1/2\delta}$. In other words, consistency of the regulated expression implies that the volume regulator L_{min} and the integral regulator δ are related and should be removed together as above.

Hamiltonian densities of Fourier modes. – The Hamiltonian densities \mathcal{H}_k (8) and \mathcal{H}_κ (10) can be related to each other by using the equations (11,18) for *positive* κ *i.e.* for $\kappa > 0$ as

$$\frac{\bar{\mathcal{H}}_\kappa}{|\kappa|} = \frac{h_\kappa}{|\kappa|} + \left(\frac{e^{2\pi\kappa/a} + 1}{e^{2\pi\kappa/a} - 1} \right) \frac{1}{\zeta(1 + 2\delta)} \sum_{r=1}^\infty \frac{\mathcal{H}_{k_r}/|k_r|}{r^{1+2\delta}}, \quad (19)$$

where $h_\kappa = \sum_{k \neq k'} [\frac{1}{2} \tilde{\pi}_k \tilde{\pi}_{-k'} F_1^\delta(k, -\kappa) F_1^\delta(-k', \kappa) + \frac{1}{2} |\kappa|^2 \tilde{\phi}_k \tilde{\phi}_{-k'} F_0^\delta(k, -\kappa) F_0^\delta(-k', \kappa)]$. The expression of h_κ involves terms which are *linear* in $\tilde{\phi}_k$'s and $\tilde{\pi}_k$'s. Hence it will dropout from vacuum expectation value.

Number operator and vacuum state. – In our analysis, so far we have used only classical aspects of the complex-valued mode functions $\tilde{\phi}_k$ and $\tilde{\pi}_k$. We now redefine these modes in terms of real-valued functions ϕ_k and π_k such that the *reality condition* of the scalar field Φ is satisfied. Hamiltonian density then becomes $\mathcal{H}_k = \frac{1}{2} \pi_k^2 + \frac{1}{2} |k|^2 \phi_k^2$ along with Poisson bracket $\{\phi_k, \pi_{k'}\} = \delta_{k,k'}$. This is the usual Hamiltonian for decoupled harmonic oscillator.

In Fock quantization, each of these modes is quantized using Schrodinger method. Corresponding creation and annihilation operators, namely \hat{a}_k^\dagger and \hat{a}_k , are used to express the field operator and to define *number density* operator $\hat{N}_k = \hat{a}_k^\dagger \hat{a}_k$. Hamiltonian density operator becomes $\hat{\mathcal{H}}_k = (\hat{N}_k + \frac{1}{2})|k|$. In polymer quantization although the operator $\hat{\mathcal{H}}_k$ exists, the notions of creation and annihilation operators are not available. Nevertheless, we note that there would be no Unruh radiation if the acceleration parameter a vanishes. So using Hamiltonian density operator we can define an alternate number density operator for Unruh particles of positive frequency $\bar{\omega} = \kappa > 0$, as seen by Rindler observer, as follows

$$\hat{N}_{\bar{\omega}} \equiv \left[\hat{\mathcal{H}}_\kappa - \lim_{a \rightarrow 0} \hat{\mathcal{H}}_\kappa \right] |\kappa|^{-1}. \quad (20)$$

This definition of number density operator reproduces the standard result for Fock quantization.

For the inertial observer we denote the vacuum state as $|0^\Theta\rangle \equiv \prod_k |0_k^\Theta\rangle$ where Θ refers to a particular quantization method namely Fock or polymer. Using the equation (19) along with the fact that $\langle 0^\Theta | \dot{\phi}_k | 0^\Theta \rangle = 0$ and $\langle 0^\Theta | \hat{\pi}_k | 0^\Theta \rangle = 0$, we express vacuum expectation value of the operator (20) as

$$\bar{N}_{\bar{\omega}} \equiv \langle 0^\Theta | \hat{N}_{\bar{\omega}} | 0^\Theta \rangle = \frac{1 - \gamma_\star}{e^{2\pi\bar{\omega}/a} - 1}, \quad (21)$$

where $E_k^0 = \langle 0^\Theta | \hat{\mathcal{H}}_k | 0^\Theta \rangle$ and $\gamma_\star = \lim_{\delta \rightarrow 0} \gamma_\star^\delta$ with

$$1 - \gamma_\star^\delta \equiv \frac{1}{\zeta(1 + 2\delta)} \sum_{r=1}^\infty \frac{\epsilon_r}{r^{1+2\delta}}, \quad \epsilon_r \equiv \frac{2E_{k_r}^0}{|k_r|}. \quad (22)$$

Fock quantization. – In Fock quantization, energy spectrum of the k^{th} oscillator is given by $\mathcal{H}_k |n_k\rangle = E_k^n |n_k\rangle$ with $E_k^n = (n + \frac{1}{2})|k|$. In this case $\epsilon_r = 1$ which then implies $\gamma_\star^\delta = 0$. So the expectation value of the

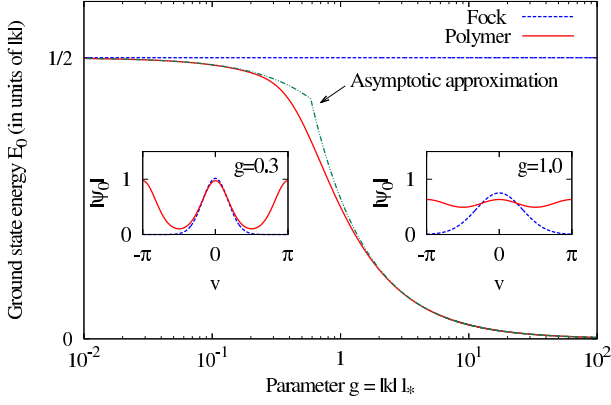


FIG. 1: The comparison of the ground state energy for different modes in Fock and polymer quantization. In the left inset panel, the modulus of the polymer vacuum ce_0 mostly follows that of the Fock vacuum, the *Gaussian*, for smaller $|v|$ ($|\pi k| \ll \lambda_*^{-1}$). On the right panel, the modulus of ce_0 differs significantly from that of the Gaussian even for smaller $|v|$. The same relative normalizations for the states have been used in both the plots.

number density operator in Fock vacuum state $|0^F\rangle$ becomes

$$\bar{N}_{\bar{\omega}} = \langle 0^F | \hat{N}_{\bar{\omega}} | 0^F \rangle = \frac{1}{e^{2\pi\bar{\omega}/a} - 1}. \quad (23)$$

The equation (23) represents *thermal spectrum for bosons* at temperature $(a/2\pi k_B)$ where k_B is the *Boltzmann constant*. This phenomenon is referred as Unruh effect.

Polymer quantization. – In polymer quantization, one represents exponentiated momentum $U_\lambda \equiv e^{i\lambda\pi_k}$ as an *elementary* operator in the kinematical Hilbert space. The classical variables ϕ_k, U_λ satisfy Poisson bracket $\{\phi_k, U_\lambda\} = i\lambda U_\lambda$. Corresponding commutator bracket is given by $[\hat{\phi}_k, \hat{U}_\lambda] = -\lambda \hat{U}_\lambda$. Operator \hat{U}_λ is not *weakly continuous* in λ hence $\hat{\pi}_k$ itself cannot be made a well-defined operator. Nevertheless, one can define an operator $\hat{\pi}_k^* = (\hat{U}_{\lambda_*} - \hat{U}_{\lambda_*}^\dagger)/2i\lambda_*$ which can be used to represent the momentum. In the limit $\lambda_* \rightarrow 0$, classically π_k^* reduces to π_k . In polymer quantization this limit however doesn't exist and λ_* is taken to be a *small but finite* scale. This scale is treated as a new dimension-full parameter of the formulation. In the context of quantum gravity, one would consider λ_* to be associated with Planck length L_p as $\lambda_* \sim \sqrt{L_p}$.

Energy eigenvalues for the k^{th} oscillator can be computed in polymer quantization as [12]

$$\frac{E_k^{2n}}{|k|} = \frac{1}{4g} + \frac{g}{2} A_n(g), \quad \frac{E_k^{2n+1}}{|k|} = \frac{1}{4g} + \frac{g}{2} B_{n+1}(g), \quad (24)$$

where $n \geq 0$, A_n and B_n are *Mathieu characteristic value functions*. Energy eigenfunctions are $\psi_{2n}(v) = ce_n(1/4g^2, v - \pi/2)$ and $\psi_{2n+1}(v) = se_{n+1}(1/4g^2, v - \pi/2)$, where $v = \lambda_* \pi_k$. The functions ce_n and se_n which are solutions to *Mathieu equation*, are referred as elliptic cosine

and sine functions [13]. The *dimensionless* parameter g is defined as $g = |k|\lambda_*^2 \equiv |k| l_*$ and it signifies the strength of polymer corrections for a given mode. For low-energy modes *i.e.* for small g , energy eigenvalues (24) become

$$\frac{E_k^{2n}}{|k|} \approx \frac{E_k^{2n+1}}{|k|} \approx n + \frac{1}{2} - \frac{(2n+1)^2 + 1}{16} g + \mathcal{O}(g^2). \quad (25)$$

Clearly, polymer quantization reproduces known results for low-energy modes. For trans-Planckian modes *i.e.* for large g , ground state energy becomes $E_k^0 = (1/4g + \mathcal{O}(g^{-3}))|k|$ which is different from the expression of low-energy modes.

For simplicity, we extend the asymptotic forms of E_k^0 for small and large g , towards $g = (2 - \sqrt{2})$ from both ends so that one asymptotic form takes over from the other asymptotic form continuously in g . In other words, we consider $\epsilon_r = (1 - r/\nu_1 r_*)$ for $r < r_*$ and $\epsilon_r = (r_*/\nu_2 r)$ for $r \geq r_*$ where $r_* = (L/l_*)(2 - \sqrt{2})/2\pi$, $\nu_1 = 2(2 + \sqrt{2})$ and $\nu_2 = 2(2 - \sqrt{2})$. It can be seen from Fig. (1) that it is a good approximation of exact ϵ_r and leads to

$$\sum_{r=1}^{\infty} \frac{\epsilon_r}{r^{1+2\delta}} = \zeta_{r_*}(1+2\delta) - \frac{\zeta_{r_*}(2\delta)}{\nu_1 r_*} + \frac{r_* \zeta(2+2\delta, r_*)}{\nu_2}, \quad (26)$$

where $\zeta_{r_*}(1+2\delta) = \sum_{r=1}^{r_*} r^{-(1+2\delta)}$, $\zeta_{r_*}(2\delta) = \sum_{r=1}^{r_*} r^{-(2\delta)}$ are *truncated zeta functions* and $\zeta(2+2\delta, r_*) = \sum_{r=r_*}^{\infty} r^{-(2+2\delta)}$ is *Hurwitz zeta function*. In the limit $\delta \rightarrow 0$, these functions are finite as long as r_* is finite. We have mentioned earlier that the limit $\delta \rightarrow 0$ forces the limit $L_{min} \rightarrow 0$. However L_{max} and the polymer scale l_* both being finite, $r_* \sim (L/l_*)$ remains finite in the limit $\delta \rightarrow 0$. Use of *zeta function identity* $\lim_{s \rightarrow 0} [s \zeta(1+s)] = 1$ then leads to $\gamma_* = 1$.

Therefore, expectation value of the number density operator (20) in polymer vacuum state $|0^P\rangle$ vanishes *i.e.*

$$\bar{N}_{\bar{\omega}} = \langle 0^P | \hat{N}_{\bar{\omega}} | 0^P \rangle = 0. \quad (27)$$

Conclusions. – We have shown that Unruh effect is *absent* in polymer quantization of (1 + 1) dimensional massless scalar field. This result is in contrast to Fock quantization. Furthermore, it demonstrates that trans-Planckian modifications can change a theoretical prediction. These trans-Planckian modifications are known to violate Lorentz invariance [12]. Therefore, the result shown here is not surprising as one does not expect such large deviation due to Lorentz invariant modifications [14]. Secondly, polymer vacuum state $|0^P\rangle$ which is represented by *zeroth order elliptic cosine* function ce_0 for each mode, remains a vacuum state even for an accelerating observer in the sense that expectation value of the number density operator in it remains zero. This is unlike the case of Fock vacuum state $|0^F\rangle$ which is represented by *Gaussian* function for each mode.

We emphasize that the result shown here is independent of the approximation that we have made for ϵ_r . Rather, it depends on the fact that $\sum_{r=1}^{\infty} \epsilon_r r^{-(1+2\delta)}$ is

finite in polymer quantization even in the limit $\delta \rightarrow 0$. This is made possible thanks to the existence of a *small but finite* scale l_* in the theory. In the context of quantum gravity the scale l_* would correspond to Planck length L_p . Finally, we note that several proposals have been made in literature to test Unruh effect in laboratory [15–17]. Based on the results shown here, we may conclude

that if experimental measurement of Unruh effect is ever possible then it can be used to either verify or rule out a theory of quantum gravity.

Acknowledgements – We would like to thank Ritesh Singh for discussions. GS would like to thank UGC for supporting this work through a doctoral fellowship.

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